Serviciencia, S. L.
Application Note
NA S04-i

Assessment of the orthogonality in Helmholtz coils

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October 2011

Summary

We describe a rather simple procedure to determine the orthogonality between the magnetic axes of two Helmholtz pairs by its mutual inductance, with a resolution of ± 0.01º or better. It allows the assessment of orthogonality errors to about ± 0.1º in sets of coils medium and small (diameter < 600 mm), and up to <± 0.05º for sets of diameter < 200 mm.

Introduction

There is a growing demand for small and tested orthogonality errors on the Helmholtz coil-sets in two or three axes. A review of the literature has not allowed us to find procedures for determining that error in this type of coils.

To our knowledge, the procedure used so far to estimate the orthogonality error is based on trigonometric calculations based on the estimated errors in the geometry of the assembly during manufacture, which are usually more or less well known. This procedure has a relatively high uncertainty and it is not well suited for a fine tuning of the geometry in order to reduce the orthogonality error to a small and known value (< ± 0.1º, for example), because of the many dimensional errors that contribute, some of which may be poorly known.

Therefore it was necessary a measurement procedure, albeit approximate, by the most direct method, for the orthogonality of the magnetic axes of any Helmholtz coil-set, also enabling tests to be performed while making corrections in the geometry of the set during the final tests, provided that the structure of the set allows such corrections.

Given the above, we have developed a procedure based on the measurement of the mutual inductance (M) of two pairs of coils, in order to determine the orthogonality of their magnetic axes. The procedure is simple and fast enough to be able to check the orthogonality while making corrections to the geometry of the assembly, to achieve a given maximum error.

Procedure fundamentals

It consists of measuring the mutual inductance (M) between two pairs of coils, using an instrument readily available as a LCR bridge with capability to measure M.

In a Helmholtz set of three axes, which we will call X, Y and Z, there are three orthogonal pairs of axis: XY, XZ and YZ, so one must measure the respective mutual inductances M_{XY}, M_{XZ} and M_{YZ} to determine the three-axis orthogonality.

The mutual inductance (M) is zero when the two axes are exactly 90º, which means a perfect orthogonality impossible in practice. When the angle (α) differs a little from 90º a small value of M can be measured, what is proportional to the difference to 90º. The value of M is maximum (M_0) when the coils are parallel (and the pairs concentric in any case), with a 0º angle between its magnetic axes. See Fig. 1.

By knowing M_0 and its variation curve to zero at 90º we can determine the value of M corresponding to any difference in angle to 90º.

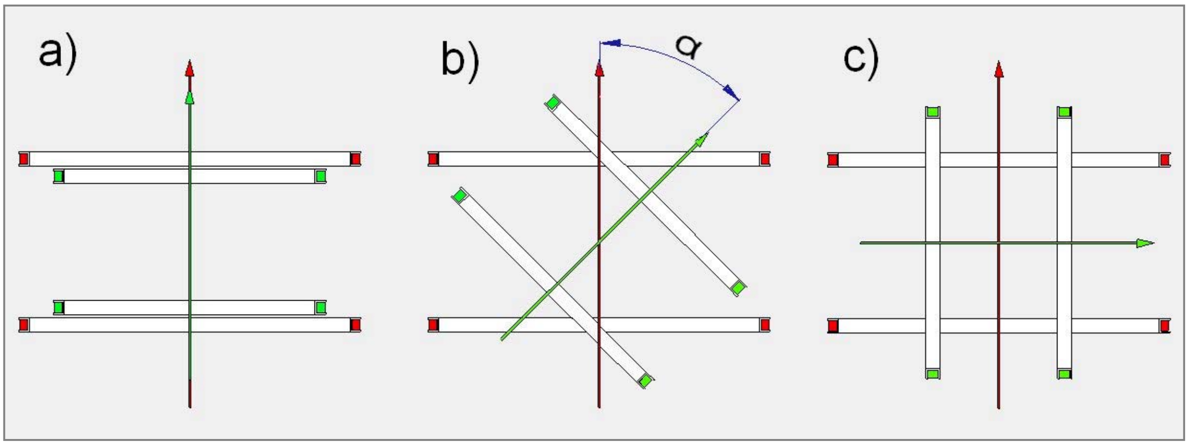

Fig. 1: a) Maximum M; b) M varies with angle α; c) Perfect orthogonality and M=0.

The misalignment of some coil in a pair also increases the value of M. Some typical misalignments that could be produced at the coil-set manufacture are shown in Fig. 2 in an exaggerated way. It is possible to diminish these defects at factory by using M as the indicator.

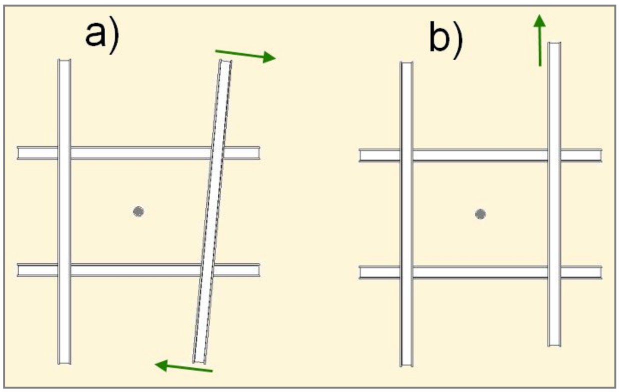

Fig. 2: Two possible coil misalignments causing an increase of M.

The two procedures we used for the determination of M_0 between different pairs of coils are described at the end.

As an approach to the variation of M with respect to the angle between the pairs, we use a variation proportional to *cos α*. A coefficient of proportionality ***a*** was also introduced to cope with the variable flux linkage in between the two coil pairs.

Thus the formula to use is:

$$M_\alpha = a \cdot M_0 \cdot \cos\alpha \qquad (1)$$

Where M_α is the mutual inductance for an angle α between the axes of the two coils, ***a*** is a proportionality coefficient and M_0 is the maximum mutual inductance, for $\alpha = 0$.

This formula is similar to the given by Grover for two circular filaments with inclined axes [1].

Measurements

To check empirically the value of M_α and get the value of ***a*** for any angle, we have used a set of coils termed here BP1 (Table 1). It is a typical coil-set, representing a large percentage of those that are built everywhere. The coils are mounted on a support as shown in Fig. 3, which allows a pair to rotate concentrically relative to the other and measure the angle α with a resolution of 0.1° and an estimated error < ±0.2°.

Fig. 3: Apparatus to measure M as a function of the angle.

Table 1 - BP1 coils used for measurements			
	X Pair	Y Pair	Z Pair
Effective diameter, ±1 mm	300 mm	266 mm	237 mm
Number of turns	83	74	66
Wire diameter	0.9 mm	0.9 mm	0.9 mm
Winding section, ±0,5 mm	10 x 8.6 mm	10 x 8.4 mm	10 x 8 mm

Since we are interested in variations of M for angles very close to 90°, we will only consider here angles between 85° and 90°. Within these limits, both the values of M_α from (1) and the measured ones have a linear relationship in practice with respect to the angle, so ***a*** is considered as constant for each pair of axes under study.

In Fig. 4 we show two curves for pairs X and Y, the calculated with (1) for ***a*** = 1 and the measured one.

Only the curves for M_{XY} are shown. The curves for M_{XZ} and M_{YZ} are similar, however the value of ***a*** should be different for each couple of coil pairs for the calculated values to fit the measured ones.

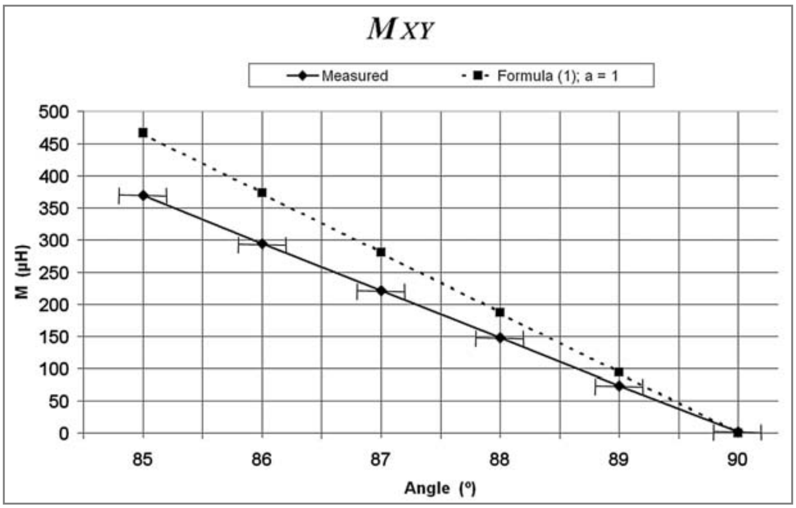

Fig. 4: Mutual inductance between *X* and *Y* pairs. Measured and calculated with (1) and ***a*** = 1.

In Table 2 are shown the values of M_0 and those of ***a*** to fit (1) to the measurements. In all three cases is ***a*** < 1.

Table 2 - Values of M_0 and *a*, for the coils BP1

	XY	*XZ*	*YZ*
M_0 ($\alpha = 0º$)	5.36 mH	3.31 mH	3.82 mH
a For 85º < α < 90º	0.79	0.92	0.79

It could be expected that the larger the difference in diameter between the pairs of coils, the closer to unity will be the value of ***a***, as for the *XZ* pairs in Table 2, because the smaller pair would occupy a more homogeneous volume of the magnetic field generated by the larger pair.

Therefore we can use (1) as a good approach to estimate the orthogonality of a typical Helmholtz coil-set, using ***a*** = 0.85 as an average value when the value of ***a*** for these specific coils has not been determined. In that case it can be also careful to use a lower value, as ***a*** = 0.75 for example, to establish the maximum margins of error.

Examples of application

Example 1: Suppose we wish to determine *M* for an orthogonality error of ± 0.2º between *XY* pairs of BP1 coils. In (1) is ***a*** = 0.79, M_0 = 5,360 µH (Table 2) and α = 89.8º. The result is M_α = 14.8 µH. So if the value of the measurement is < ±14.8 µH, then the orthogonality error of the coils is < ±0.2º.

Example 2: Suppose we wish to determine *M* for some orthogonality error between two pairs of coils whose coefficient ***a*** has not been determined. After obtaining M_0 by any of the methods mentioned in below (or someone else), we may assume ***a*** = 0.75 putting ourselves in the worst case. For added security we could still use ***a*** = 0.50. The rest would be as in Example 1.

Some drawbacks

The procedure works well in practice for coils up to about 600 mm in diameter. Above this size some difficulties become important. One difficulty appears when increasing too much the length of the cables for the LCR bridge, what can impair the measurements. Another difficulty is due to the interaction of the coils with the near ferromagnetic material in the measuring room, which affects the magnetic field generated by the LCR bridge and the inductance of the coils, what can produce wrong measurements. That interaction increases with the diameter of the coils. All around the coils being measured should be free from large ferromagnetic objects. As AC current is used for measuring, nearby objects with a high electrical conductivity could also affect.

Determination of M_0

We have determined the values of M_0 for BP1 coils by two different methods.

The first method was direct measurement of M_0, for what a suitable stand was made to mount the three coil pairs concentrically and with $\alpha = 0$, in order to measure M_0 between them.

The second method consisted of computer simulation using Finite Element Analysis (FEA). The mutual inductance M_0 can be extracted from the results of the used FEA program. An advantage of this method is that this can provide results for coils not yet constructed.

The resulting M_0 values were similar in both cases, measurement and calculation with FEA, with differences <2.5% between these, mainly attributable to small errors in the geometry and assembly of the coils in one case and minor errors in the FEA model in the other.

Conclusions

The described procedure is practical and accurate enough for checking purposes in coil-sets manufacture. Further work and more powerful 3-D FEA or analytical tools, would enable to improve the formula (1) to predict M with higher precision for angles close to 90º, for a wide variety of Helmholtz coil formats.

References

[1] Grover FW (2009), "Inductance calculations, working formulas and tables", Mineola, NY: Dover Publications, p. 193.
